

Sample questions for KAUST Mathematics Competition Category A

1. Since $\sqrt{x^2} = |x|$ and $\sqrt{3} > 3/2$, we have

$$\sqrt{\left(\frac{3}{2} - \sqrt{3}\right)^2} + \sqrt{\left(\sqrt{3} - \frac{1}{2}\right)^2} = \left|\frac{3}{2} - \sqrt{3}\right| + \left|\sqrt{3} - \frac{1}{2}\right| = \sqrt{3} - \frac{3}{2} + \sqrt{3} - \frac{1}{2} = 2\sqrt{3} - 2$$

2. Let the field's area be X and the time they need is T . Notice that Ahmad's speed is $X/240$, while Khalid's is $X/360$. Then we need to solve $(\frac{X}{240} + \frac{X}{360})T = X \implies T = 144$.

3. Notice that $a^2 + 2ab + 4b^2 + 2^{10} = 2^{10} + 2^{10} + 2^{10} + 2^{10} = 4 \cdot 2^{10} = 2^{12}$

4. The lowest possible sum of six distinct positive integers is $1 + 2 + 3 + 4 + 5 + 6 = 21$. As the desired sum is just one higher, the only possibility is $1 + 2 + 3 + 4 + 5 + 7 = 22$, as adding one to any other integer produces two integers of the same value. Then the answer is $1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 7 = 840$.

5. The first 3 steps are as follows:

$$6 \rightarrow 3 \rightarrow 5 \rightarrow 6.$$

6. If P is the intersection point between the semi-circle and the segment BD , and O is the midpoint of AB then the area can be computed in the following way:

$$\text{ShadedArea} = [\triangle ABD] - [\text{Sector}(OPA)] - [\triangle OPB] = \frac{12^2}{2} - \frac{1}{4} \cdot 6^2\pi - \frac{6 \cdot 6}{2} = 54 - 9\pi.$$

7. The interior angle of a regular pentagon is 108° , so $w = 108, x = 108/2 = 54, y = 108/3 = 36, z = 108/4 = 27$. Then, $w + x + y + z = 108 + 54 + 36 + 27 = 225$.

8. Ahmad will pick one book or magazine in $7 + 3$ ways, and one pen or pencil in $5 + 3$ ways. So the total number of ways is $10 \cdot 8 = 80$.

9. First we observe that a segment connecting centers of two cells has length $\sqrt{5}$ if and only if the two cells are at the opposite corners in a 2×3 or a 3×2 rectangle. Therefore, in each 2×3 or 3×2 rectangle, there are two such segments (one connecting the top left and the bottom right, one connecting the top right and the bottom left). Clearly there is an equal number of 2×3 and 3×2 rectangles, by rotation. Thus, the total number of segments is 4 times the number of 3×2 rectangles. To count these rectangles, we consider the position of the top left corner since the rest of the rectangle is then determined. The top left corner must be in the top left 9×10 subgrid, since otherwise the 3×2 rectangle would go out of the bounds of the 11×11 grid. As such, the number of 3×2 rectangles is $9 \cdot 10 = 90$, and the number of desired segments is $4 \cdot 90 = 360$.

10. The total cost is $12 \cdot 20 = 240$ riyals. After 2 friends cancel, each of the remaining 10 friends must pay $240/10 = 24$ riyals, which is 4 riyals more than what they had to pay.