Sample questions for KAUST Mathematics Competition Category B

1.

$$\frac{2^{-2}+3^0}{(-0.5)^{-2}-5\cdot 2^{-2}+\left(\frac{2}{3}\right)^{-2}}+3.75=\frac{\frac{1}{4}+1}{(\frac{1}{-2})^{-2}-5\cdot\frac{1}{4}+\left(\frac{3}{2}\right)^{2}}+3.75=$$

$$=\frac{\frac{5}{4}}{(-2)^2-\frac{5}{4}+\frac{9}{4}}+3.75=\frac{\frac{5}{4}}{5}+4.75=\frac{1}{4}+3.75=4.$$

2. The first time number n appears in the sequence is at the position: $[1+2+...+(n-1)]+1=\frac{(n-1)n}{2}+1$, and the last position is $[1+2+3+\cdots+(n-1)+n]=\frac{n(n+1)}{2}$.
$(n-1)n<2\cdot 5000\le n(n+1)\Rightarrow n^2=10,000\Rightarrow n=100$

3. Notice that we can substitute $x=3$ directly into the expression for $H(x)$:

$$\begin{aligned}H(3)&=[F(4)]+[G(2)]\\&=(4^2+4+1)+(2^2-2+1)\\&=(21)+(3)=24\end{aligned}$$

4. Notice that $y^2+5=x^3+1$ then $\frac{y^2+5}{x+1}=\frac{x^3+1}{x+1}=x^2-x+1$. Hence, the expression is equal to $x^2-y+1=5+1=6$.

5. It is clear that the height of the cylinder is 2 and the radius of the circle is $\sqrt{2}$. Hence, the desired volume is

$$\pi(\sqrt{2})^2\cdot 2-2^3=4\pi-8.$$

6. $x+70=35+100$, so $x=65$.

7. To maximize the value of $a-2c$, we need a to be the largest side and c the shortest side. Hence, $a\ge b\ge c\ge 1$. From the triangle inequality,

$$a<b+c=2025-a\quad\Rightarrow\quad a\le 1012.$$

Then $a-2c\le 1012-2\cdot 1=1010$. Equality occurs for $(a,b,c)=(1012,1012,1)$.

8. We are looking for the smallest number $s\ge 555$ of the form $s=27x+36y$, where x and y stand for the numbers of buses of the first and the second type, respectively. Since the greatest common divisor of 27 and 36 is 9, s has to be a multiple of 9. The smallest multiple of 9 which is greater or equal to 555 is 558 and since $558=27\cdot 18+36\cdot 2$, we conclude that the number of empty seats is $558-555=3$.

9. Let $n+1=k^2\Rightarrow n=k^2-1$. From (d): $n+4=k^2+3$ is three-digit, so $100\le k^2+3\le 999$, giving $10\le k\le 31$.
From (c): $n+3=k^2+2\equiv 0 \pmod{11}\Rightarrow k^2\equiv 9 \pmod{11}$. Thus $k\equiv \pm 3 \pmod{11}$. From

(b): $n+2=k^2+1$ is prime, so k is even. In the range $10 \le k \le 31$, possible k are $14, 30$. We check (b):

$$k=14 \Rightarrow n=195, \quad n+2=197 \text{ prime},$$
$$k=30 \Rightarrow n=899, \quad n+2=901=17\cdot 53 \text{ not prime}.$$

Hence $n=195$ and sum of digits of $3n=585$ is $5+8+5=18$.

10. Since $y \in [-11, 11]$, we obtain the following

- If $y=\pm 11$, then $x=0 \Rightarrow 2\cdot 1$ pairs.
- If $y=\pm 10$, then $x=0, \pm 1 \Rightarrow 2\cdot 3$ pairs.
- If $y=\pm 9$, then $x=0, \pm 1, \pm 2 \Rightarrow 2\cdot 5$ pairs.

$\ldots$

- If $y=\pm 1$, then $x=0, \pm 1, \ldots, \pm 10 \Rightarrow 2\cdot 21$ pairs.
- If $y=0$, then $x=0, \pm 1, \ldots, \pm 11 \Rightarrow 23$ pairs.

Summing all these numbers(adding first-last,...), we get

$$2(1+3+\ldots+19+21+23)-23=2(6\cdot 24)-23=265.$$

11. The sums divisible by 5 are $(5, 10)$. The probabilities for each are $\frac{4}{36}, \frac{3}{36}$ respectively. Thus, the answer is their sum which is $\frac{7}{36}$

12. When the game ends, the winner has 10 points, while all the other players have at most 9 points, which sums up to $10+4\cdot 9=46$.