Sample Questions for KAUST Mathematics Competition
Final Round, Seniors Track

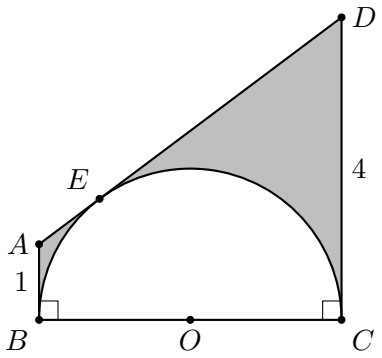
1. What is the least positive integer n such that $n \cdot 1! \cdot 4! \cdot 4! \cdot 7!$ is a perfect square?
Note: For a positive integer k, $k!$ is the product of all positive integers from 1 to k. That is, $k! = k \cdot (k-1) \cdot (k-2) \cdots 3 \cdot 2 \cdot 1$.

2. Hatem selects three different integers at random from the set $\{-5, -4, -3, \cdots, 4, 5\}$. What is the probability that the sum of the selected numbers is 0?

3. Find all triples x, y, z satisfying both equations

$$x^2 + 4y^2 + 2(x - 2y) + 2 = 0 \quad \text{and} \quad 2xy + 4yz + zx - 4 = 0.$$

4. In the diagram below, $ABCD$ is a trapezoid with $AB \parallel CD$, $BC \perp AB$, $AB = 1$, and $CD = 4$. A semicircle with center O and diameter $\overline{BC}$ is tangent to $\overline{AD}$ at E. Calculate the area of the shaded region.



5. Determine the largest possible integer k such that $300^9 - 3^9$ is divisible by 3^k.

6. Four polygons – one equilateral triangle and three other congruent regular polygons, all with unit side length – are placed on a table. Every two polygons share exactly one side, and no two overlap. What is the perimeter of the shape formed by these polygons?

7. Let $x_1, x_2, x_3, \cdots, x_{2026}$ be a sequence of real numbers that satisfy the relation:

$$x_{i+1} = \frac{i - x_i}{i}$$

for all $i = 1, 2, \ldots, 2025$. Find the sum:

$$S = x_1 + 2x_2 + 3x_3 + \cdots + 2024x_{2024} + 2025x_{2025} + 2025x_{2026}.$$

8. Several distinct integers, each greater than 20, are written on a board. The product of the smallest and largest of them equals the sum of the remaining numbers. What is the minimum possible number of integers on the board?